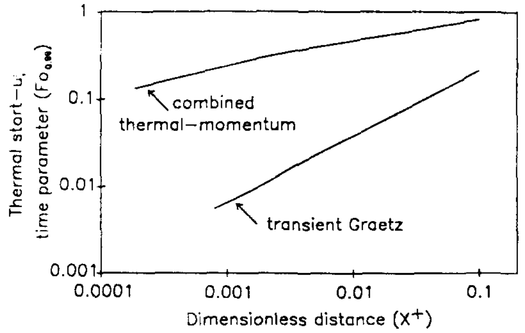

FIG. 4. Thermal start-up times for the transient Graetz and the combined thermal–momentum start-up problems.

the results presented in Fig. 4, equations (12) and (13) are proposed for estimating the start-up time in the transient Graetz and the combined thermal–momentum problems, respectively:

transient Graetz

$$Fo_{0.99} = 1.2(X^+)^{3/4}; \tag{12}$$

combined thermal–momentum

$$Fo_{0.99} = 1.5(X^+)^{1/4}. \tag{13}$$

4. CONCLUSIONS

A solution method is outlined for the combined thermal–momentum start-up problem. By assuming the fluid properties to be constant, the momentum equation is decoupled from the energy equation. The energy equation is expressed in a finite difference form and solved numerically with the velocity profile calculated from the analytical solution of the momentum equation. Results indicate that the start-up time for the combined thermal–momentum start-up problem is much larger than that for the transient Graetz problem. Simple correlations are proposed for the start-up time for the two transient laminar heat transfer problems.

Acknowledgement—We thank Mr T. Papathanasiou for helpful discussions. Financial support was provided by the Natural Sciences and Engineering Research Council of Canada (NSERC).

REFERENCES

1. R. Siegel, Heat transfer for laminar flow in ducts with arbitrary time variations in wall temperature, *Trans. ASME, J. Appl. Mech* **82E**, 241–249 (1960).
2. M. Perlmutter and R. Siegel, Two-dimensional unsteady incompressible laminar duct flow with a step change in wall temperature, *Int. J. Heat Mass Transfer* **3**, 94–107 (1961).
3. S. Kakac and Y. Yener, Transient laminar forced convection in ducts. In *Low Reynolds Number Flow Heat Exchangers* (Edited by S. Kakac, R. K. Shah and A. E. Bergles), pp. 205–227. Hemisphere, New York (1983).
4. T. F. Lin, K. H. Hawks and W. Leidenfrost, Transient thermal entrance heat transfer in laminar pipe flows with step change in pumping pressure, *Wärme- und Stoffübertr.* **17**, 201–209 (1983).
5. R. M. Cotta and M. N. Ozisik, Transient forced convection in laminar channel flow with stepwise variations of wall temperature, *Can. J. Chem. Engng* **64**, 734–742 (1986).
6. R. M. Fithen and N. K. Anand, Finite-element analysis of conjugate heat transfer in axisymmetric pipe flows, *Numer. Heat Transfer* **13**, 189–203 (1988).
7. Y. Joshi and B. Gebhart, Transient response of a steady vertical flow subject to a change in surface heating rate, *Int. J. Heat Mass Transfer* **31**, 743–756 (1988).
8. R. Creff and P. Andre, A numerical model for an unsteady developing flow and its associated heat transfer problems. In *Computational Techniques of Fluid Flow* (Edited by C. Taylor and T. A. Johnson), pp. 219–253. Pineridge Press, Swansea (1986).
9. P. Szymanski, Quelques solutions exactes des equations de l'hydrodynamique de fluide visqueux dans le cas d'un tube cylindrique, *J. Math. Pures Appliquees* **11**, Series 9, 67–107 (1932).
10. G. S. Patience, Numerical solutions for laminar forced convection and fluid flow in pipes, M.Sc. thesis, University of Calgary, Calgary, Canada (1987).
11. G. S. Patience and A. K. Mehrotra, Laminar start-up flow in short pipe lengths, *Can. J. Chem. Engng* **67**, 883–888 (1989).

Int. J. Heat Mass Transfer. Vol. 33, No. 9, pp. 2053–2055, 1990
Printed in Great Britain

0017-9310/90 $3.00 + 0.00

Moisture and temperature dependence of the moisture diffusivity

J. DRCHALOVÁ, O. KAPIČKOVÁ and F. VODÁK
Faculty of Civil Engineering, Czech Technical University, Thákurova 7, 166 29 Praha 6, Czechoslovakia

and

T. KLEČKA
Institute of Civil Engineering, Czech Technical University, Šolínova 7, 166 08 Praha 6, Czechoslovakia

(*Received* 25 *October* 1989)

THE DEPENDENCE of the moisture diffusivity $D(u, T)$ on the moisture content u and on the temperature T is important not only in technical applications of porous materials, but it can also yield valuable information on the mechanism of the unsaturated water flow in capillary-porous media. It is usually assumed [1] that in the transport of the liquid phase the surface tension σ plays the role of the driving force and the viscosity η of the liquid is responsible for energy dissipation that leads to a quasi-stationary character of water flow. Dimensional analysis yields the following expression

NOMENCLATURE

$D(u,T)$	moisture diffusivity [$m^2\,s^{-1}$]
$f(r)$	distribution curve of pore radii [m^{-1}]
$F(r)$	integrated distribution function of pore radii
r	radius of the pore [m]
$r(u)$	largest radius of the pore filled by water at moisture content u [m]
$Q(u)$	correction factor, equation (5)
$R(u)$	geometrical factor [m]
$R_0(u)$	geometrical factor [m]
t	time [s]
T	temperature [°C]
u	moisture content
$u(x)$	moisture distribution along the sample
$x(u)$	inverse function to $u(x)$ [m]
$W(T)$	function characterizing properties of water [$m\,s^{-1}$].
Greek symbols	
$\eta(T)$	dynamical viscosity [$m^{-1}\,kg\,s^{-1}$]
θ	wetting angle
$\sigma(T)$	surface tension [$kg\,s^{-2}$]
$\tau(u)$	tortuosity.

for the moisture diffusivity $D(u,T)$:

$$D(u,T) = W(T)R(u),\; W(T) = \sigma(T)\cos\theta/4\eta(T) \quad (1)$$

where θ is the wetting angle. The moisture diffusivity D is factorized into a product of two functions: $W(T)$, characterizing the transport properties of water, and the geometrical factor $R(u)$ characterizing the system of pores which is connected with the pore distribution function $f(r)$ and with the tortuosity $\tau(u)$ by the relation

$$R(u) = R_0(u)/\tau^2(u) \quad (2)$$

where

$$R_0(u) = \frac{1}{r^2(u)f(r(u))}\int_0^{r(u)} r^2 f(r)\,\mathrm{d}r. \quad (3)$$

This formula was derived [1, 2] under the assumption that pores of increasing size are consecutively filled up during water intake. More specifically, it is assumed that at moisture content u pores of radii $0 < r < r(u)$ are filled, while those with radii $r > r(u)$ are empty. Obviously, the function $r(u)$ is increasing and it is connected with $f(r)$ by the relation

$$u = u_s F(r(u)),\; F(r) = \int_0^r f(r')\,\mathrm{d}r' \quad (4)$$

where u_s is the highest possible value of u that corresponds to saturation, and the function $F(r)$ is a primitive function of $f(r)$. The pore distribution function $f(r)$ is normalized to unity: $F(\infty) = 1$.

The aim of the present work was to verify (i) the temperature dependence of D as given by equation (1), and (ii) the functional form, equation (3), of the geometrical factor $R(u)$.

All the measurements were carried out on samples in the form of rods (300 mm long with cross-section 40 × 10 mm) made of ceramics (soft bisque composed of 35% Al_2O_3, 44% kaoline and 21% feldspar). Due to the low baking temperature (700°C) most of the pores remained open. This material is particularly suitable for our purposes because it is perfectly homogeneous and it has only small pores, which ensures that the water transport is not influenced by gravitation. We determined the moisture diffusivity D by using the Matano method [3] for all accessible values of the moisture content u, i.e. for $0 < u < u_s$, $u_s = 0.194$ corresponds to saturation. The samples were held in the horizontal position during measurements in order to minimize the gravitational effects. One end of the sample was fed by water, its surfaces were carefully insulated by water- and vapour-tight epoxy resin, and the other end was exposed to dry air. The moisture distribution $u(x)$ along the sample was determined by the gravimetric method with high accuracy. The samples were thermostatically controlled during all measurements in order to keep constant temperature and humidity of the ambient air. The measurements were carried out at four temperatures, $T = 4$, 27, 40 and 60°C. A typical moisture distribution $u(x)$ along the sample is shown in Fig. 1.

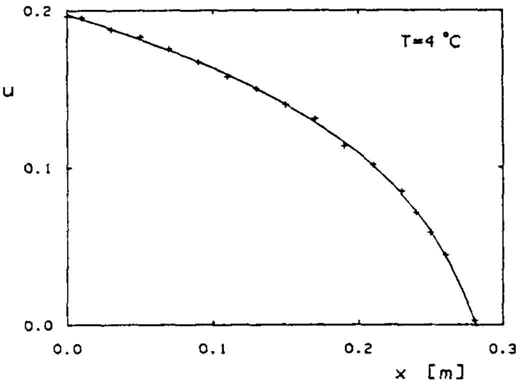

Fig. 1. Moisture distribution along the sample at $T = 4$°C and $t = 432\,900$ s. The experimental values are denoted by crosses and their fit by a cubic polynomial is shown by the full line.

The method of Matano [3] is based on the solution of the non-linear diffusion equation via Boltzmann's transformation [4]. It gives the moisture diffusivity in the form

$$D(u,T) = -\frac{x'(u)}{2t}\int_{u_0}^{u} x(u')\,\mathrm{d}u' \quad (5)$$

where $x(u)$ is the inverse function to $u(x)$, and t the time needed to reach the moisture distribution $u(x)$, provided that $u(x) = u_0$ for all x at $t = 0$. To evaluate our experimental data we approximated the function $x(u)$ by a cubic polynomial the coefficients of which were found by a least-square fit. The moisture diffusivity D was then calculated using equation (5). The resulting curves are shown in Fig. 2.

The geometrical factor $R(u)$ is calculated from equation (1). We employed the tabulated values of $\eta(T)$ [5] and $\sigma(T)$ [6] and for simplicity we set $\cos\theta = 1$ for all temperatures. It is a good approximation since the wetting angle of water on silicates is close to 0° and the change of $\sigma(T)$ between 4 and 60°C is only by a factor of 0.883 [6]. The resulting curves are almost identical and are shown in Fig. 3. Note that $R(u)$ is a polynomial of sixth degree in u if $x(u)$ is approximated by a cubic polynomial.

We determined $f(r)$ experimentally using the high-pressure mercury porosimeter Carlo Erba AG 60. The measurements were carried out at pressures ranging from 0.1 to 100 MPa, which corresponds to pore radii between 7500 and 7.5 nm. The integrated pore distribution function $F(r)$ is shown in Fig. 4. We calculated the geometrical factor $R_0(u)$ according to equation (3) using the measured pore distribution function $f(r)$ (dashed line in Fig. 3). As no smoothing of experimental data has been carried out and we have used the piecewise linear interpolation for the function $F(r)$, the calculated geometrical factor $R_0(u)$ shows several jumps.

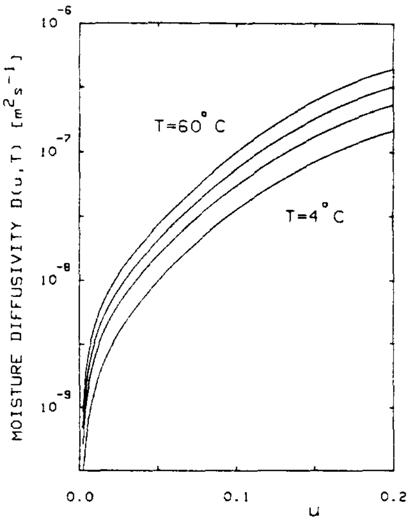

FIG. 2. The dependence of the moisture diffusivity $D(u, T)$ on the moisture content u at temperatures $T = 4$, 27, 40 and 60°C. The lowest curve corresponds to 4°C, the highest one to 60°C.

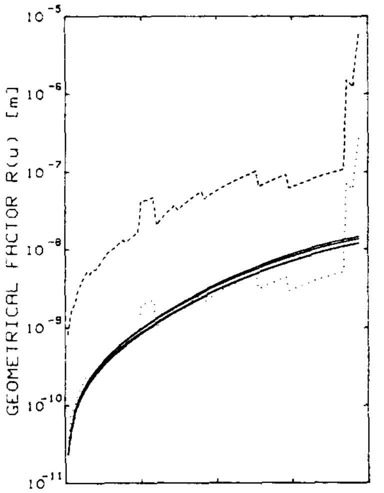

FIG. 3. The geometrical factors $R(u)$ calculated from equation (1) at four different temperatures (full lines). Also shown is the geometrical factor $R_0(u)$ calculated according to equation (2) from the experimentally determined pore distribution function $f(r)$ (dashed line), and $R_0(u)/Q(u)$ (dotted line).

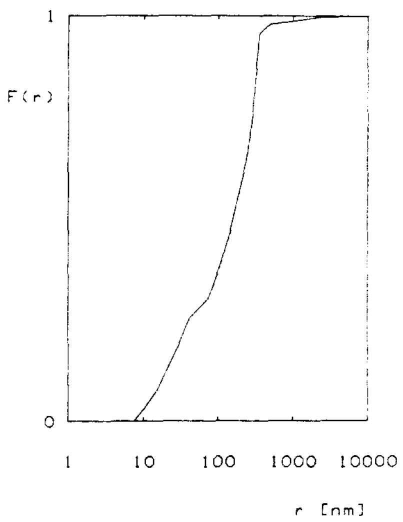

FIG. 4. Experimentally determined integrated pore distribution function $F(r)$.

Its general shape however, closely resembles that of $R(u)$, calculated from $D(u, T)$. It differs by a correction factor

$$Q(u) = R_0(u)/R(u) = \tau^2(u)\cos\theta \qquad (6)$$

which is almost constant for all values of the moisture content. Note that independence of $Q(u)$ on u corresponds to a constant value of tortuosity in the whole range of pore radii. By means of a least-square fit we have found $Q = 20.97$, which seems to be a reasonable value, because for $\cos\theta = 1$ it yields tortuosity $\tau = 4.58$. There is, of course, some uncertainty in the value of the wetting angle. For example, for $\theta = 40°$ the tortuosity is $\tau = 4.01$.

We have studied the temperature dependence of the moisture diffusivity D and the functional form of its dependence on the moisture content. The main results of the present work may be summarized as follows.

(1) The temperature dependence of $D(u, T)$ is determined with a good accuracy by the temperature dependence of the surface tension $\sigma(T)$ and viscosity $\eta(T)$ according to equation (1).

(2) The dependence of $D(u, T)$ on the moisture content u is given by the geometrical factor $R(u)$, which is directly connected with the distribution function $f(r)$ of pore radii and with the tortuosity $\tau(u)$.

(3) Our results confirm the assumptions on the mechanism of water transport in the liquid phase in capillary-porous media.

REFERENCES

1. A. W. Lykow, *Transporterscheinungen in Kapillarporösen Körpern*. Akademie, Berlin (1958).
2. A. Belghith et B. Le Fur, Étude de la diffusivité hydraulique dans un milieu poreux en fonction de la teneur en eau et de la température, *Comp. Rend. Acad. Sci. Paris* **268A**, 349–352 (1969).
3. C. Matano, On the relation between the diffusion-coefficients and concentration of solid metals (the nickel-copper system), *Jap. J. Phys.* **8**, 109–113 (1933).
4. J. Crank, *The Mathematics of Diffusion*. Clarendon Press, Oxford (1975).
5. *CRC Handbook of Chemistry and Physics* (Edited by R. C. Weast), 56th Edn. CRC Press, Cleveland (1975).
6. Landolt-Börnstein, *Zahlenwerte un Funktionen*, II/3, 6. Auflage (Edited by K. Schäfer and E. Lax). Springer, Berlin (1956).